Technical Notes

Optimization and Visualization of Functions

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A numerical technique which results in computer-generated contour maps is described This technique has been employed extensively in the optimization of functions of several variables It has also proved to be an invaluable aid to subsequent exact investigation of a function's extrema by conventional numerical search techniques A computational scheme that may be used to generate stereographic views of a mathematical function is also described The resulting perspective and/or "3-D" views enhance one's ability to understand the nature and structure of complicated mathematical functions

Introduction

NUMEROUS optimization techniques are currently being employed to locate maxima and minima of functions of several variables Although conventional methods (e g , steepest descent, calculus of variations) are capable of isolating extrema, they are incapable of providing concise, easily understood information concerning the nature and structure of complicated functions

Recent studies concerning optimum two-impulse orbital transfer inspired the use of computer-generated contour maps as a means of optimization [1-3] It was also found that three-dimensional objects or functions are more easily understood and studied when presented as perspective and/or stereographic views [4] The computational methods used to produce contour maps and stereographic views are outlined in this note

Contouring Procedure

Any set of data which is a function of two variables may be systematically studied by presenting the information as a contour map A large amount of data can thus be compressed into a concise "readable" form The techniques developed here will contour data arranged in a rectangular array (rows and columns) The array of points $Z(x,y)$ is systematically contoured by considering rectangles having four adjacent points for vertices (Fig 1) Each rectangle is then divided into two triangles by choosing that diagonal which has minimum ΔZ between its end points

Since each triangle represents a plane, the problem reduces to one of determining the lines of intersection of the contour planes and the individual triangles This may be accomplished by performing linear interpolation between the vertices of the triangle to locate points where individual contour planes intersect each side These points are then paired, and individual segments of contour lines are drawn between them Figure 2 presents a typical contour map which was generated using this technique A 20 × 20 array of points was utilized for this example Contouring the function required several seconds of IBM 7090 time

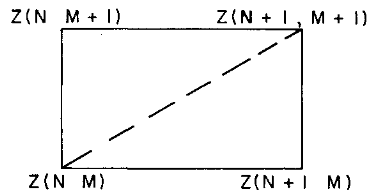

Fig 1 Geometry of contouring procedure

This technique involves several approximations which influence the fidelity of the final contour map (e g , short straight line segments rather than curves, planes rather than interpolating functions) Typically, these defects should be of no concern The increased cost of using a more sophisticated method of interpolation justifies the use of these approximations Furthermore, the goal here is analysis of the function rather than exact or artistic representation

A number of properties of the function are immediately apparent from a cursory examination of the contour map Four local optima (the regions having nearly circular contours) are easily observed One may also identify saddle points, steep "walls," etc

Although many of the function's details are apparent in the contour map, probably the most important information presented concerns the gross behavior and structure of the function Conventional numerical optimization techniques are not capable of presenting this type of information A number of experiments using numerical techniques have

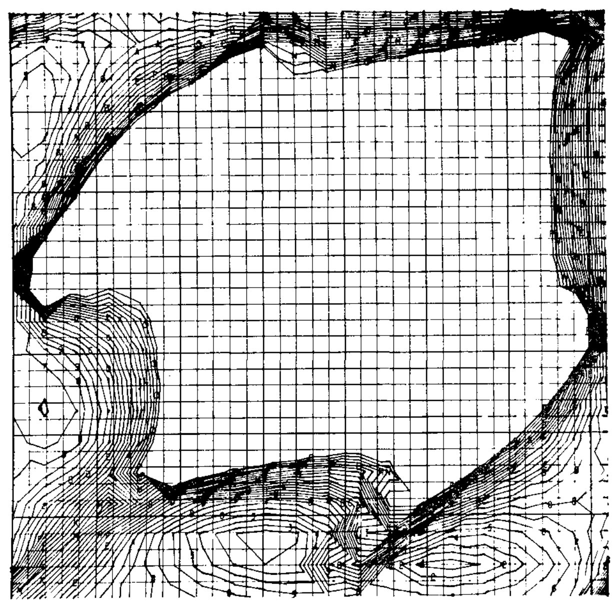

Fig 2 Contour map of an optimum impulse function

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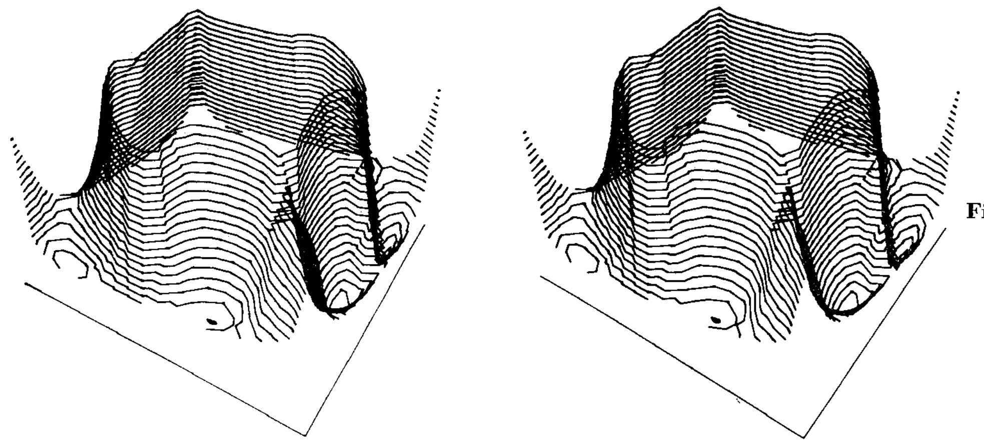

Fig 3 Stereographic views of impulse function

verified the results of contouring In all instances, the contouring methods have proved invaluable to the subsequent detailed numerical investigation by conventional optimization methods

Perspective Views of a Function

Computer mechanization of the proper transformations for a perspective view is a straightforward vector process which duplicates what the eye sees when viewing an object from a given position First, the given geometrical object or function must be described in a convenient coordinate system For any given position of an observer in the same coordinate system, it is possible to construct a transformation matrix which maps points of the object into a projection plane The computational procedure employed is described briefly below

One must first select a point which lies in the projection plane and defines one end of the line of sight (This point should be near the center of the object) The observer's position in the coordinate system allows definition of a line-of-sight vector **O** normal to the projection plane and directed toward the observer Coordinate vectors **U** and **V** in the projection plane are chosen so that (**U**, **V**, **O**) forms an orthogonal set

Points of the object (defined by a three-dimensional vector, **A**) may now be mapped into the projection plane by considering the line established by (**A** − **O**) and its intersection with the projection plane This point of intersection defines a two-dimensional vector in the projection plane with components along the **U** and **V** coordinate vectors Line segments whose end points are generated by this method may then be systematically plotted in the projection plane to form a perspective view of the object (Fig 3) The method is rapid and economical—perspective plotting of 1000 line segments requires less than 1 sec of IBM 7090 time The actual plotting of the function is accomplished using an S-C 4020 CRT

Stereographic Views of a Function

Two properly generated perspective views may be placed in a stereoscopic viewer to produce a three-dimensional presentation of an object Computationally, these views are generated by varying the line of sight to allow for the normal separation of the eyes of an observer Figure 3 presents left and right frames of a stereo-pair A magnifying stereoscope, similar to those used for interpreting aerial survey photographs, may be employed to yield a three-dimensional presentation of the function

Ordinarily, the function which appears in Fig 3 would be presented in the form of a contour map (Fig 2) The average person is unable to visualize the details of the function from the contour map alone A single perspective view proves helpful for this purpose, but the data are still difficult to interpret By contrast, few people find it difficult to understand the function when it is presented in a stereoscope In practice, insight gained through function visualization has greatly simplified subsequent detailed investigation of complicated functions by numerical search techniques

References

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Back-Scattering Properties of Moon and Earth at *X* Band

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Introduction

THE back-scattering cross section per unit area of lunar surface $\sigma_0(\phi)$ is presented here,‡ knowledge of this functional dependence being necessary for the design of vehicle-borne radars measuring altitude and velocity very accurately, for purposes of navigation over the moon Analogous results for the earth are also given, since it is thought that the designers of radar for earth-bound vehicles might then be able to estimate easily, by comparison, the requirements for Moon navigation radars

The information is exhibited in Fig 1 § Smoothed-out functions are presented in this figure, which give no indica-

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‡ The argument ϕ is the angle of incidence shown in Fig 2

§ The experimental results for the earth were obtained by Campbell[1] In the case of the moon, the angular dependence of σ_0 [in the form of $(1/k)$ $\sigma_0(\phi)$, where k is an arbitrary constant] was obtained by Evans and Pettingill[2] An estimate of the absolute spectrum $\sigma_0(\phi)$ was then provided by Evans,[3] based on the value of σ_m given by Eq (2)